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# A Novel Computing Approach for Third Order Boundary Layer Equation

(Kaedah Pengiraan Baru bagi Persamaan Lapisan Sempadan Tertib Ketiga)

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ABSTRACT

*This paper proposed an efficient modification of homotopy perturbation transform method (HPTM), namely modified homotopy perturbation transform method (MHPTM) for the solution of third order boundary layer equation on semi-infinite domain. The technique was based on the application of Laplace transform to boundary layers in fluid mechanics. The nonlinear terms can be easily handled by the use of He's polynomials. The Pade´ approximants, that often show superior performance over series approximations, were effectively used in the analysis to capture the essential behavior of the boundary layer equation on infinity. We then conduct a comparative study between the MHPTM and the existing results with the help of third order boundary layer equation. The results obtained indicated that the MHPTM was effective and promising.*

*Keywords: He's polynomials; modified Adomian decomposition method (MADM); modified Laplace decomposition method (MLDM); Pade´ approximants; third order boundary layer equation*

ABSTRAK

*Makalah ini mencadangkan pengubahsuaian yang lebih cekap untuk kaedah jelmaan usikan homotopi (HPTM), iaitu kaedah jelmaan usikan homotopi terubah suai (MHPTM) untuk menyelesaikan persamaan lapisan sempadan peringkat tiga dalam domain semi-terhingga. Teknik ini adalah berasaskan penggunaan jelmaan Laplace bagi lapisan sempadan dalam mekanik bendalir. Sebutan tak linear boleh ditangani dengan mudah menggunakan polinomial He. Penghampiran Padé yang sentiasa menunjukkan prestasi yang baik terhadap penghampiran-penghampiran siri digunakan secara cekap dalam analisis untuk memperoleh telatah penting persamaan lapisan sempadan di ketakterhinggaan. Kajian bandingan antara MHPTM dengan keputusan sedia ada dengan bantuan persamaan lapisan sempadan peringkat tiga juga dilakukan. Keputusan yang diperoleh menunjukkan yang MHPTM adalah berkesan dan meyakinkan.*

*Kata kunci: Kaedah penguraian Adomian terubah suai (MADM); kaedah penguraian Laplace terubah suai (MLDM); penghampiran Padé; persamaan lapisan sempadan peringkat tiga; polinomial He*

## INTRODUCTION

Many analytical and numerical techniques have been developed by various scientists to cope with the nonlinear boundary layer problems arising in fluid mechanics. Laminar boundary layer flow of an incompressible fluid has several important engineering applications such as the aerodynamic extrusion of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along liquid film condensation process, glass and polymer industries. Various ways have been proposed recently to deal with these nonlinear problems such as the Adomian decomposition method (Abdou 2005; Adomian 1994; Biazar 2005, 2006; Biazar et al. 2010; El-Wakil & Abdou 2007; Šmarda & Archalousova 2010; Soliman & Abdou 2008; Wazwaz 1997, 2006), the Laplace decomposition method (Khan 2009; Khan & Austin 2010; Khan & Faraz 2011; Khan et al. 2011), the homotopy perturbation method (HPM) (Biazar & Ghanbari 2012; Khan et al. 2011; Slota 2010, 2011; Turkyilmazoglu 2011; Xu 2007), the variational iteration method (Abdou 2007; El-Wakil & Abdou 2008; Soliman 2009, 2011; Turkyilmazoglu 2011) and the homotopy analysis method (Biazar & Ghanbari 2012; Turkyilmazoglu 2011).

In this paper, we proposed a new approach to develop an approximate solution for the fluid mechanics problems. But unfortunately, in providing or developing methods, we only focus on its strong points and ignore the barriers that lead us away from the true solution. As numerical methods are meant for problems which true solutions are hard to obtain, we should have a logical analysis of the method for a variety of problems so that there is greater agreement between the facts and the solutions. For example, the Laplace transform is an elementary but useful technique for solving linear ordinary differential equations that is widely used by scientists and engineers for tackling linearized models. However, the Laplace transform is totally incapable of handling nonlinear equations because of the difficulties caused by the nonlinear terms. To overcome this shortcoming, Khan and Wu (2011) suggest a combined form of the Laplace transform method with the homotopy perturbation method which is developed for analytic treatment of the nonlinear partial differential equations, namely homotopy perturbation transform method (HPTM) that will facilitate the calculations. This technique basically illustrates how the Laplace transform can be used to approximate the solutions of the nonlinear differential equations by manipulating the decomposition method which was first introduced by Adomian (1994). The method is very well suited to physical problems since it does not require unnecessary linearization and other restrictive methods and assumptions which may change the problem being solved, sometimes seriously.

The objectives of this paper were three-fold: first, to introduce the new analytical method for finding the analytical solution of boundary layer equation which primarily lie in its ability to avoid the unnecessary calculations of other iteration methods; second, our aim in this article was to compare the results with solutions to the existing ones (Khan & Faraz 2011; Xu 2007; Wazwaz 2006) and third, to extend and modified our previous approach proposed in Khan and Wu (2011) on semi-infinite domain.

## MODIFIED HOMOTOPY PERTURBATION TRANSFORM METHOD (MHPTM)

To illustrate the basic idea of this method, we consider a general nonlinear non-homogeneous 3rd order nonlinear ordinary differential equation with initial condition of the form:

$$f''' + b_1(x)f'' + b_2(x)f' + b_3(x)f = g(f), \tag{1}$$

$$f(0) = \alpha, f'(0) = \beta, \ f''(0) = \gamma,$$

where $\alpha, \beta, \gamma \in C(R_+)$, $i = 1,2,3$ and $g(f)$ is the nonlinear source term.

According to homotopy perturbation transform method (Khan et al. 2011), first we apply Laplace transform (denoted throughout this paper by $L$) on both sides of equation (1):

$$s^3L[f] - s^2\alpha - s\beta - \gamma + L[b_1(x)f''] + L[b_2(x)f'] + L[b_3(x)f] = L[g(f)], \tag{2}$$

which gives:

$$L[f] = \frac{\alpha}{s} + \frac{\beta}{s^2} + \frac{\gamma}{s^3}L[g(f)] - \frac{1}{s^3}L[b_1(x)f'' + b_2(x)f' + b_3(x)f]. \tag{3}$$

Operating with Laplace inverse on both sides of (3) gives:

$$f = G(x) + \frac{1}{s^3}L[g(f)] - L^{-1}\left[\frac{1}{s^3}L[b_1(x)f'' + b_2(x)f' + b_3(x)f]\right]. \tag{4}$$

where $\boldsymbol{G(x)}$ represents the term arising from the prescribe initial condition. We now assume that the solution can be expressed in an infinite series in the form:

$$f = \sum_{n=0}^{\infty} p^n f_n, \tag{5}$$

the nonlinear term can be decomposed as:

$$g(f) = Nf = \sum_{n=0}^{\infty} p^n H_n, \tag{6}$$

for some He's polynomials $H_n$ (Ghorbani 2009; Zayed & Rahman 2012) that are given by:

$$H_n = \frac{1}{n!}\frac{d^n}{dp^n}\left[N\left(\sum_{i=0}^{\infty} p^i f_i\right)\right]_{p=0}, \quad n = 0,1,2,3 \ldots$$

Substituting (6) and (5) in (4), we get:

$$\sum_{n=0}^{\infty} p^n f_n = G(x) + p\left(\begin{array}{l} L^{-1}\left[\frac{1}{s^3}L\left[\sum_{n=0}^{\infty} p^n H_n\right]\right] \\ -L^{-1}\left[\frac{1}{s^3}L\left[\begin{array}{l} b_1(x)\sum_{n=0}^{\infty} p^n f_n'' \\ b_2(x)\sum_{n=0}^{\infty} p^n f_n' \\ +b_3(x)\sum_{n=0}^{\infty} p^n f_n \end{array}\right]\right] \end{array}\right). \tag{7}$$

Comparing co-efficient of like powers of $p$, the following approximations are obtained:

$$\begin{aligned} p^0 &: f_0 = G(x), \\ p^1 &: f_1 = L^{-1}\left[\frac{1}{s^3}L[H_0]\right] - L^{-1}\left[\frac{1}{s^3}L\left[\begin{array}{l} b_1(x)f_0'' + \\ b_2(x)f_0' + \\ b_3(x)f_0 \end{array}\right]\right], \\ p^2 &: f_2 = L^{-1}\left[\frac{1}{s^3}L[H_1]\right] - L^{-1}\left[\frac{1}{s^3}L\left[\begin{array}{l} b_1(x)f_1'' + \\ b_2(x)f_1' + \\ b_3(x)f_1 \end{array}\right]\right], \end{aligned} \tag{8}$$

The modified method suggests that the function $\boldsymbol{G(x)}$ defined above in (8) be decomposed into two parts, namely $G_0(x)$ and $G_1(x)$. Such that:

$$\boldsymbol{G(x) = G_0(x) + G_1(x)}. \tag{9}$$

Instead of (8), we suggest the following new iteration procedure:

$$p^0 : f_0 = G_0(x),$$

$$p^1: f_1 = G_1(x) + L^{-1}\left[\frac{1}{s^3}L[H_0]\right] - L^{-1}\left[\frac{1}{s^3}L\left[b_1(x)f_0'' + b_2(x)f_0' + b_3(x)f_0\right]\right],$$

$$p^2: f_2 = L^{-1}\left[\frac{1}{s^3}L[H_1]\right] - L^{-1}\left[\frac{1}{s^3}L\left[b_1(x)f_1'' + b_2(x)f_1' + b_3(x)f_1\right]\right], \quad (10)$$

PADÉ APPROXIMANTS

Padé approximants constitute the best approximation of a function by a rational function of a given order. Developed by Henri Padé, Padé approximants often provide better approximation of a function than Taylor Series truncating does and they may still work in cases in which the Taylor Series does not converge. For these reasons, Padé approximants are used extensively in computer calculations and it is now well known that these approximants have the advantage of being able to manipulate polynomial approximation into the rational functions of polynomials. Padé approximants is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function (Baker 1975). The $[L/M]$ Padé approximant to a formal power series is given by $[L/M] = P_L(x)/Q_M(x)]$, where $P_L(x)$ is a polynomial of degree at most $L$ and $Q_M(x)$ is a polynomial of degree at most $L$. Without loss of generality, we can assume $Q_M(0)$ to be 1. Furthermore, $P_L(x)$ and $Q_M(x)$ have no common factors. This means that the formal power series $A(x)$ equal the $[L/M]$ approximant through $L + M + 1$ terms. It is a well known fact that Padé approximants will converges on the entire real axis if $f(\eta)$ is free of singularities on the entire real axis. More importantly, the diagonal approximants are the most accurate approximants, therefore we will construct on diagonal approximants. Using the boundary condition $f(\infty) = 0$, the diagonal approximants $[M/M]$ vanish if the coefficients of numerator vanish with the highest power in the $\eta$. Choosing the coefficients of the highest power of $\eta$ equal to zero, we get a polynomial equations in $\eta$ which can be solved very easily by using the built in utilities in the most manipulation languages such as Matlab and Mathematica.

THIRD ORDER BOUNDARY LAYER EQUATION

Let us consider the following nonlinear third order boundary layer problem arising in fluid mechanics (Khan & Faraz 2011; Xu 2007; Wazwaz 2006):

$$f''' + (n-1)ff''' - 2n(f')^2 = 0, \quad n > 0,$$
$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0. \quad (11)$$

In this work, we will examine $f''(0) = \alpha < 0$. By applying the MHPTM subject to the initial conditions, we have:

$$L[f] = \frac{s+\alpha}{s^3} + \frac{1}{s^3}L\left[2n(f')^2 - (n-1)ff''\right]. \quad (12)$$

The inverse Laplace transform applied to (12) results in:

$$f(\eta) = \eta + \frac{\alpha\eta^2}{2} + L^{-1}\left[\frac{1}{s^3}L\left[2n(f')^2 - (n-1)ff''\right]\right]. \quad (13)$$

In view of homotopy perturbation transform method (Khan et al. 2011), we have:

$$\sum_{m=0}^{\infty} p^m f_m(\eta) = G(\eta) + p\left(L^{-1}\left[\frac{1}{s^3}L\left[2n\left(\sum_{m=0}^{\infty} H_{1m}(\eta)\right) - (n-1)\left(\sum_{m=0}^{\infty} H_{2m}(\eta)\right)\right]\right]\right), \quad (14)$$

In the above equation $H_{1m}(\eta)$ and $H_{2m}(\eta)$ are the He's polynomials (Ghorbani 2009; Zayed & Rahman 2012) that represent the nonlinear terms. Through the modified method the function $G(\eta)$ can be written as:

$$G(\eta) = f_0(\eta) + f_1(\eta) = \eta + \frac{\alpha\eta^2}{2}. \quad (15)$$

By this consideration, we first set modified recursive relations in the form:

$$p_0 : f_0(\eta) = \eta, \quad (16)$$

$$p^1 : f_1(\eta) = \frac{\alpha\eta^2}{2} + L^{-1}\left[\frac{1}{s^3}L\left[2n(H_{10}) - (n-1)(H_{20})\right]\right],$$
$$p^2 : f_2(\eta) = L^{-1}\left[\frac{1}{s^3}L\left[2nH_{11}(\eta) - (n-1)H_{21}(\eta)\right]\right], \quad (17)$$

Using $f_0(\eta) = \eta$ in (17), the other components are:

$$f_1(\eta) = \frac{\alpha\eta^2}{2} + \frac{\eta\eta^3}{3},$$
$$f_2(\eta) = \frac{\alpha(3n+1)\eta^4}{24} + \frac{n(n+1)\eta^5}{30},$$
$$f_3(\eta) = \frac{\alpha^2(3n+1)\eta^5}{120} + \frac{\alpha(19n^2+18n+3)\eta^6}{720} + \frac{n(2n^2+2n+1)\eta^7}{315},$$
$$f_4(\eta) = \frac{\alpha^2(27n^2+42n+11)\eta^7}{5040} + \frac{\alpha(167n^3+297n^2+161n+15)\eta^8}{40320} + \frac{n(13n^3+38n^2+23n+6)\eta^9}{22680}. \quad (18)$$

The series solution is given as:

$$
\begin{aligned}
f_3(\eta) = {} & \eta + \frac{\alpha\eta^2}{2} + \frac{n\eta^3}{3} + \frac{\alpha(3n+1)\eta^4}{24} \\
& + \left(\frac{1}{30}n^2 + \frac{1}{40}n\alpha^2 + \frac{1}{30}n\right)n^5 \\
& + \left(\frac{19}{720}n^2\alpha + \frac{1}{240}\alpha + \frac{1}{40}n\alpha\right)\eta^6 \\
& + \left(\frac{1}{120}n^2\alpha + \frac{1}{315}n + \frac{2}{315}n^3\right. \\
& \left. + \frac{11}{5040}\alpha^2 + \frac{3}{560}n^2\alpha^2 + \frac{2}{315}n^2\right)\eta^7 \\
& + \left(\frac{11}{40320}\alpha^3 + \frac{33}{44880}n^3\alpha + \frac{3}{4480}\alpha^3 n^2\right. \\
& \left. + \frac{23}{5760}n\alpha + \frac{1}{2688}\alpha + \frac{167}{40320}n^3\alpha + \frac{1}{960}\alpha^3 n\right)\eta^8 \\
& + \begin{pmatrix} \frac{1}{3780}n + \frac{527}{362880}n^3\alpha^2 + \frac{19}{11340}n^3 + \frac{709}{362880}n\alpha^2 \\ + \frac{23}{8064}n^2\alpha^2 + \frac{23}{22680}n^2 + \frac{13}{22680}n^4 + \frac{43}{120960}\alpha^2 \end{pmatrix}\eta^9 + \ldots
\end{aligned}
\tag{19}
$$

## CONCLUSION

In this work, we have carefully developed a modified homotopy perturbation transform method (MHPTM). The MHPTM overcomes the difficulties arising in the homotopy perturbation transform method established by Khan et al. (2011). The proposed technique was employed without any linearization and discretization. The solution through the modified method highly depends upon the choice of $G_0(x)$ and $G_1(x)$. Moreover, the powerful diagonal Pade´ approximants were applied to get a better understanding of the solution behavior. The obtained series solution is combined with the diagonal Pade´ approximants to handle the boundary condition at infinity. The results of MHPTM were compared to MADM, MLDM and HPM as shown in Tables 1 and 2. The results of the four methods have very closed agreement with each other. The modified version is also valid for other nonlinear boundary layer equations, and this paper can be used as a standard paradigm for other applications.

TABLE 1. Comparison of the numerical value of $\alpha = f''(0$ obtained by MHPTM with MADM and MLDM

| $n$ | Padé approximants | Present method | MADM (Wazwaz 2006) | MLDM (Khan & Faraz 2011) |
|---|---|---|---|---|
| 0.2 | [2/2] | –0.3872983347 | –0.3872983347 | –0.3872983347 |
| | [3/3] | –0.3821533832 | –0.3821533832 | –0.3821533832 |
| | [4/4] | –0.3819153845 | –0.3819153845 | –0.3819153845 |
| | [5/5] | –0.3819148088 | –0.3819148088 | –0.3819148088 |
| | [6/6] | –0.3819121854 | –0.3819121854 | –0.3819121854 |
| 0.3 | [2/2] | –0.5773502692 | –0.5773502692 | –0.5773502692 |
| | [3/3] | –0.5615999244 | –0.5615999244 | –0.5615999244 |
| | [4/4] | –0.5614066588 | –0.5614066588 | –0.5614066588 |
| | [5/5] | –0.5614481405 | –0.5614481405 | –0.5614481405 |
| | [6/6] | –0.561441934 | –0.561441934 | –0.561441934 |
| 0.4 | [2/2] | –0.6451506398 | –0.6451506398 | –0.6451506398 |
| | [3/3] | –0.6397000575 | –0.6397000575 | –0.6397000575 |
| | [4/4] | –0.6389732578 | –0.6389732578 | –0.6389732578 |
| | [5/5] | –0.6389892681 | –0.6389892681 | –0.6389892681 |
| | [6/6] | –0.6389734794 | –0.6389734794 | –0.6389734794 |
| 0.6 | [2/2] | –0.8407967591 | –0.8407967591 | –0.8407967591 |
| | [3/3] | –0.8393603021 | –0.8393603021 | –0.8393603021 |
| | [4/4] | –0.8396060478 | –0.8396060478 | –0.8396060478 |
| | [5/5] | –0.8395875381 | –0.8395875381 | –0.8395875381 |
| | [6/6] | –0.8396056769 | –0.8396056769 | –0.8396056769 |
| 0.8 | [2/2] | −1.007983207 | −1.007983207 | −1.007983207 |
| | [3/3] | −1.007796981 | −1.007796981 | −1.007796981 |
| | [4/4] | −1.007646828 | −1.007646828 | −1.007646828 |
| | [5/5] | −1.007646828 | −1.007646828 | −1.007646828 |
| | [6/6] | −1.007792100 | −1.007792100 | −1.007792100 |

TABLE 2. Comparison of the numerical value of $\alpha = f''(0)$ obtained by MHPTM with MADM, MLDM and HPM

| $n$ | Present Method | MADM (Wazwaz 2006) | MLDM (Khan & Faraz 2011) | HPM (Xu 2007) |
|---|---|---|---|---|
| 4 | −2.483954032 | −2.483954032 | −2.483954032 | −2.5568 |
| 10 | −4.026385103 | −4.026385103 | −4.026385103 | −4.0476 |
| 100 | −12.84334315 | −12.84334315 | −12.84334315 | −12.8501 |
| 1000 | −40.65538218 | −40.65538218 | −40.65538218 | −40.6556 |
| 5000 | −104.8420672 | −104.8420672 | −104.8420672 | –90.9127 |

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REFERENCES

Abdou, M.A. 2005. Adomian decomposition method for solving the telegraph equation in charged particle transport. *Journal of Quantitative Spectroscopy and Radiative Transfer* 95: 407-414.

Abdou, M.A. 2007. On the variational iteration method. *Physics Letters A* 366: 61-68.

Adomian, G. 1994. *Solving Frontier Problems of Physics: The Decomposition Method*. Boston: Kluwer Academic Publication.

Baker, G.A. 1975. *Essentials of Pade´ Approximants*. London: Academic Press.

Biazar, J. 2005. Solution of systems of integral-differential equations by Adomian decomposition method. *Applied Mathematics and Computation* 168: 1232-1238.

Biazar, J. 2006. Solution of the epidemic model by Adomian decomposition method. *Applied Mathematics and Computation* 173: 1101-1106.

Biazar, J., Porshokuhi, M.G. & Ghanbari, B. 2010. Extracting a general iterative method from an Adomian decomposition method and comparing it to the variational iteration method. *Computers and Mathematics with Applications* 59: 622-628.

Biazar, J. & Ghanbari, B. 2012. The homotopy perturbation method for solving neutral functional-differential equations with proportional delays. *Journal of King Saud University (Science)* 24: 33-37.

Biazar, J. & Ghanbari, B. 2012. HAM solution of some initial value problems arising in heat radiation equations. *Journal of King Saud University (Science)* 24: 161-165.

El-Wakil, S.A. & Abdou, M.A. 2007. New applications of Adomian decomposition method. *Chaos, Solitons & Fractals* 33: 513-522.

El-Wakil, S.A. & Abdou, M.A. 2008. New applications of variational iteration method using Adomian polynomials. *Nonlinear Dynamics* 52: 41-49.

Ghorbani, A. 2009. Beyond Adomian's polynomials: He polynomials. *Chaos, Solitons & Fractals* 39: 1486-1492.

Khan, Y. 2009. An effective modification of the Laplace decomposition method for nonlinear equations. *International Journal of Nonlinear Sciences and Numerical Simulation* 10: 1373-1376.

Khan, Y. & Austin, F. 2010. Application of the Laplace decomposition method to nonlinear homogeneous and non-homogeneous advection equations. *Zeitschriftfuer Naturforschung A* 65: 849-853.

Khan, Y. & Faraz, N. 2011 Application of modified Laplace decomposition method for solving boundary layer equation. *Journal of King Saud University (Science)*: 115-119.

Khan, Y., Madani, M., Yildirim, A., Abdou, M.A. & Faraz, N. 2011. A new approach to Van der Pol's Oscillator Problem. *Zeitschriftfuer Naturforschung A* 66: 620-624.

Khan, Y., Wu, Q., Faraz, N. & Yildirim, A. 2011. The effects of variable viscosity and thermal conductivity on a thin film flow over a shrinking/stretching sheet. *Computers and Mathematics with Applications* 61: 3391-3399.

Khan, Y. & Wu, Q. 2011. Homotopy perturbation transform method for nonlinear equations using He's polynomials. *Computers and Mathematics with Applications* 61: 1963-1967.

Slota, D. 2010. The application of the homotopy perturbation method to one-phase inverse Stefan problem. *International Communications in Heat and Mass Transfer* 37: 587-592.

Slota, D. 2011. Homotopy perturbation method for solving the two-phase inverse Stefan problem. *Numerical Heat Transfer Part A: Applications* 59: 755-768.

Šmarda, Z. & Archalousova, O. 2010. Adomian decomposition method for certain singular initial value problems II. *Journal of Applied Mathematics* 3: 91-98.

Soliman, A.A. & Abdou, M.A. 2008. The decomposition method for solving the coupled modified KdV equations. *Mathematical and Computer Modelling* 47: 1035-1041.

Soliman, A.A. 2009. On the solution of two-dimensional coupled Burgers' equations by variational iteration method. *Chaos, Solitons and Fractals* 40: 1146-1155.

Soliman, A.A. 2011. A numerical simulation of modified Boussinesq and seven-order Sawada-Kotara equations. *International Journal of Numerical Methods for Heat and Fluid Flow* 21: 320-330.

Turkyilmazoglu, M. 2011. Some issues on HPM and HAM methods: A convergence scheme. *Mathematical and Computer Modelling* 53: 1929-1936.

Turkyilmazoglu, M. 2011. An optimal variational iteration method. *Applied Mathematics Letters* 24: 762-765.

Turkyilmazoglu, M. 2011. An optimal analytic approximate solution for the limit cycle of Duffing-van der Pol equation. *Journal of Applied Mechanics, Transactions ASME* 78: 021005.

Wazwaz, A.M. 1997. A study on a boundary-layer equation arising in an incompressible fluid. *Applied Mathematics and Computation* 87: 199-204.

Wazwaz, A.M. 2006. The modified decomposition method and Pade´ approximants for a boundary layer equation in unbounded domain. *Applied Mathematics and Computation* 177: 737-744.

Xu, L. 2007. He's homotopy perturbation method for a boundary layer equation in unbounded domain. *Computers and Mathematics with Applications* 54: 1067-1070.

Zayed, E.M.E. & Rahman, H.M.A. 2012. On using the He's polynomials for solving the nonlinear coupled evolution equations in Mathematical Physics. *WSEAS Transactions on Mathematics* 11: 294-302.

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